① Standard Form

$A \times 10^n$

$1 \leq A \leq 10$ & n can be +ve or -ve

Example

Express in standard form:

a) $321000 = 3.21 \times 10^5$

b) $0.000678 = 6.78 \times 10^{-4}$

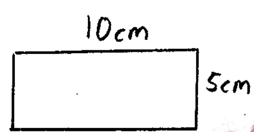
② Prime number

Memorise all prime numbers from 2 to 71.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71

③ Upper & Lower Bound

Example



Each of the length is measured correct to the nearest centimetre

Find:

(a) the upper bound for the perimeter &

(b) the lower bound for the perimeter.

Answer

(a) Upper bound ⇒ round all reading up by 0.5 cm.

10cm ⇒ 10.5cm

5 cm ⇒ 5.5cm

Perimeter $= 10.5 + 10.5 + 5.5 + 5.5$

$= 32$cm

b) Lower bound ⇒ round all reading down by 0.5cm.

10 cm ⇒ 9.5cm

5cm ⇒ 4.5cm

Perimeter $= 9.5 + 9.5 + 4.5 + 4.5$

$= 38$cm

④ Direct & Inverse Proportion

Example 1

x is directly proportional to y.

When $y = 10$, $x = 5$

Find x when $y = 20$.

Answer

$x = ky$

$5 = k(10)$

$k = \frac{5}{10} = \frac{1}{2}$

$x = \frac{1}{2}y$

When $y = 20$,

$x = \frac{1}{2}(20)$

$= 10$

Example 2

x is inversely proportional to y.

When $y = 10$, $x = 2$

Find x when $y = 30$.

Answer

$x = \frac{k}{y}$

$2 = \frac{k}{10}$

$k = 2 \times 10$

$= 20$

$x = \frac{20}{y}$

When $y = 30$

$x = \frac{20}{30}$

$= \frac{2}{3}$

⑤ Percentage

Example

Express 64 as a percentage of 80.

Answer

$\frac{64}{80} \times 100\% = 80\%$

⑦ Simple & Compound Interest

Simple interest $(I) = \frac{PRT}{100}$

P = principal, R = rate, T = time

Example 1

Calculate the interest owed if a man borrows \$300 from a bank charging 2% simple interest per month for 3 months?

$P = 300$, $R = 2\%$ $T = 3$

$I = \frac{300 \times 2 \times 3}{100}$

$= \$18$

Compound Interest

$A = P\left(1 + \frac{R}{100}\right)^n$

A = total amount after time, n

P = principal

R = rate

n = time

Example 2

Caculate the total amount owed if a man borrows \$300 from a bank charging 2% compound interest per month for 3 months?

$P = 300$, $R = 2$, $n = 3$

Total amount owed $= P\left(1 + \frac{R}{100}\right)^n$

$= 300\left(1 + \frac{2}{100}\right)^3$

$= \$318.36$

⑧ Gradient of a straight line

Gradient $= \frac{y_1 - y_2}{x_1 - x_2}$

Example

Calculate the gradient of a line that passes through point A(-2, -1) & B(4, 2)

Answer

A(-2, -1) B(4, 2)

x_1, y_1 x_2 y_2

Gradient $= \frac{-1 - 2}{-2 - 4}$

$= \frac{-3}{-6}$

$= \frac{1}{2}$

⑨ Equation of a line

$y = mx + c$

① find the gradient, m.

② find the y-intercept, c.

Example

Find the equation of a line that passes through A(-2, -1) & B(4, 2)

Answer

Eqn of a line $\Rightarrow y = mx + c$

$m = \frac{1}{2}$ (found above)

$y = \frac{1}{2}x + c$

To find c, sub in point A.

$-1 = \frac{1}{2}(-2) + c$

$-1 = -1 + c$

$c = 0$

$y = \frac{1}{2}x + 0$

$\Rightarrow y = \frac{1}{2}x$

⑩ Midpoint of 2 given points

$$\boxed{\text{Midpoint} = \left(\frac{x_1 + x_2}{2}\right), \left(\frac{y_1 + y_2}{2}\right)}$$

Example

Find the midpoint of P(-2, 8) & Q (4, -4)

Answer

P (-2 , 8) — x_1, y_1 Q (4 , -4) — x_2, y_2

$$\text{Midpoint} = \left(\frac{-2+4}{2}, \frac{8+(-4)}{2}\right)$$

$$= (1, 2)$$

⑪ Length between 2 points

$$\boxed{\text{Length} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}$$

Example

Find the distance between P(-2, 8) & Q (4, -4)

Answer

P (-2, 8) — x_1, y_1 Q (4, -4) — x_2, y_2

$$\text{Distance} = \sqrt{(-2-4)^2 + (8-(-4))^2}$$

$$= \sqrt{6^2 + 12^2}$$

$$= \sqrt{180}$$

$$= 13.4 \text{ units.}$$

⑫ Function

Example

$f(x) = 4x + 1 \qquad g(x) = x^3 + 1$

$\boxed{f(2) \Rightarrow \text{sub } x = 2 \text{ \& solve for } f(x)}$

$$f(2) = 4(2) + 1$$
$$= 8 + 1$$
$$= 9$$

$\boxed{fg(x) \Rightarrow \text{sub } x = g(x)}$

$$fg(x) = 4(x^3 + 1) + 1$$
$$= 4x^3 + 4 + 1$$
$$= 4x^3 + 5$$

$\boxed{f^{-1}(x) \Rightarrow \text{let } y = f(x) \text{ \& make } x \text{ the subj.}}$

$$\text{Let } y = f(x) = 4x + 1$$
$$y = 4x + 1$$
$$4x = y - 1$$
$$x = \frac{y-1}{4}$$
$$f^{-1}(x) = \frac{x-1}{4}$$

⑬ Indices

$\boxed{a^m \times a^n = a^{m+n}}$

Example: $3x^5 \times 4x^3 = 12x^{5+3}$
$= 12x^8$

$\boxed{a^m \div a^n = a^{m-n}}$

Example: $24x^7 \div 6x^3 = 4x^{7-3}$
$= 4x^4$

$\boxed{a^0 = 1}$

Example: $24x^7 \div 3x^7 = 8x^{7-7}$
$= 8x^0$
$= 8(1)$
$= 8$

$$\boxed{(a^m)^n = a^{m \times n} = a^{mn}}$$

Example: $(3x^2)^4 = 3^4 x^{2\times4}$
$= 81x^8$

$$\boxed{(a \times b)^n = a^n \times b^n}$$

$$\boxed{\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}}$$

$$\boxed{a^{-n} = \frac{1}{a^n}}$$

Example: $x^2y^5 \div x^7y^3 = x^{2-7}y^{5-3}$
$= x^{-3}y^2$
$= \frac{1}{x^3}y^2$
$= \frac{y^2}{x^3}$

$$\boxed{\begin{array}{l} a^{\frac{1}{n}} = \sqrt[n]{a},\ n \neq 0 \\ a^{\frac{m}{n}} = \sqrt[n]{a^m},\ n \neq 0 \end{array}}$$

Solving Eqn involving Indices

$3^x \times 3^2 = 81$
$3^{x+2} = 81$
$3^{x+2} = 3^4$
$x+2 = 4$
$x = 4-2$
$= 2$

⑭ Solving Quadratic Eqn

- By factorisation

Example: $x^2 - x - 6 = 0$
$(x-3)(x+2) = 0$
$x-3 = 0 \quad x+2 = 0$
$x = 3 \quad x = -2$

x	-3	$-3x$
x	2	$2x$
x^2	-6	$-x$

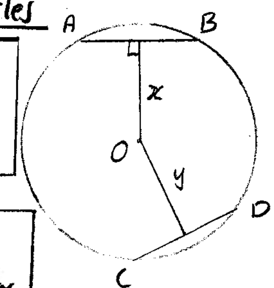
- By formula

*When question says "give your answers correct to 2 decimal places.", USE FORMULA *

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

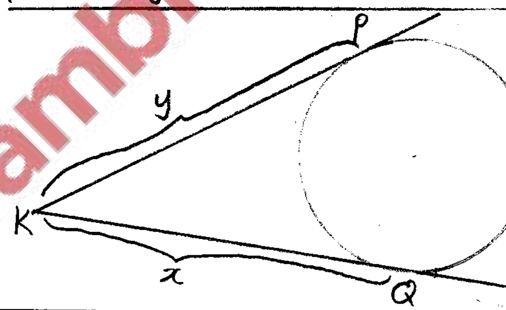
⑮ Symmetry Properties of Circles

a) Equal chords are equidistant from centre
If $AB = CD \Rightarrow x = y$



b) Perpendicular bisector of chord passes through centre

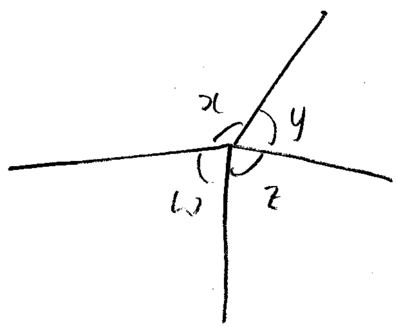
c) Tangents from an external point are equal in length $\Rightarrow KP = KQ \Rightarrow x = y$



Paper 1 Qn

⑯ Angle Properties

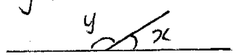
a) Angles at a point $= 360°$



$\angle w + \angle x + \angle y + \angle z = 360°$

b) **Angles on a straight line = 180°**

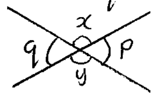
$\angle x + \angle y = 180°$



c) **Vertically opp. angles are equal.**

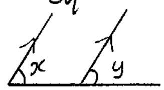
$\angle x = \angle y$

$\angle p = \angle q$



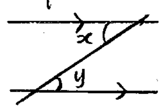
d) **Corresponding angles are equal. (F)**

$\angle x = \angle y$



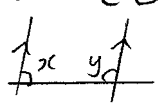
e) **Alternate angles are equal (Z)**

$\angle x = \angle y$



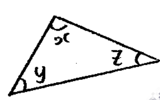
f) **Interior angles = 180° (U)**

$\angle x = \angle y$



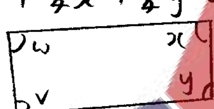
g) **Angles in a Δ = 180°**

$\angle x + \angle y + \angle z = 180°$



Angles in a quadrilateral = 360°

$\angle w + \angle v + \angle x + \angle y = 360°$

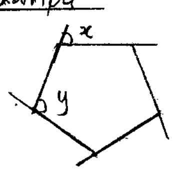


h) **Polyons & their angles**

For regular polygon with n sides, ext $\angle = \frac{360°}{n}$

For regular polygon with n sides, int. $\angle = 180° - \frac{360°}{n}$

Example



For a 5-sided polygon, $n = 5$

ext. $\angle = \angle x = \frac{360}{5} = 72°$

int. $\angle = \angle y = 180° - 72°$
$= 108°$

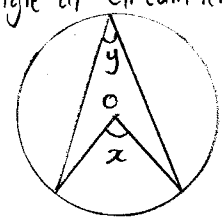
i) **Irregular Polygon & their angles**

Total ext. $\angle$s $= 360°$

Total interior $\angle$s $= (n-2) \times 180°$

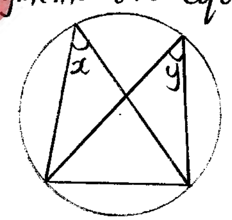
j) **Angle at centre = 2× angle at circumference**

$\angle x = 2 \times \angle y$



k) **Angles in the same segments are equal**

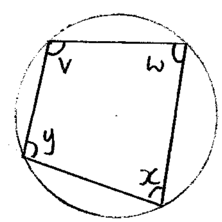
$\angle x = \angle y$



l) **Opp. $\angle$s in a cyclic quadrilateral = 180°**

$\angle v + \angle x = 180°$

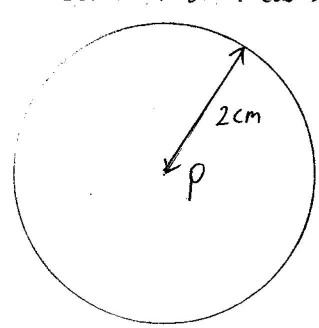
$\angle y + \angle w = 180°$



17 Locus

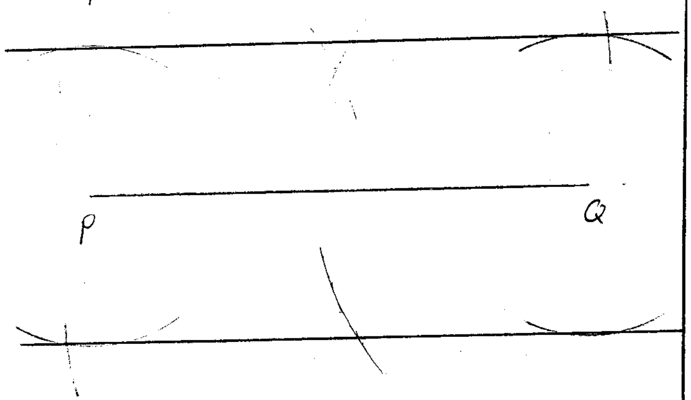
a) **Given distance from a given point**

Example: Construct a locus 2cm from P.



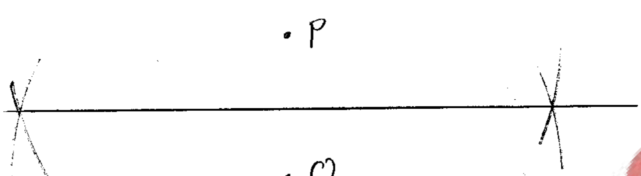
b) Given distance from a given line

Example: construct a locus 2cm from line PQ



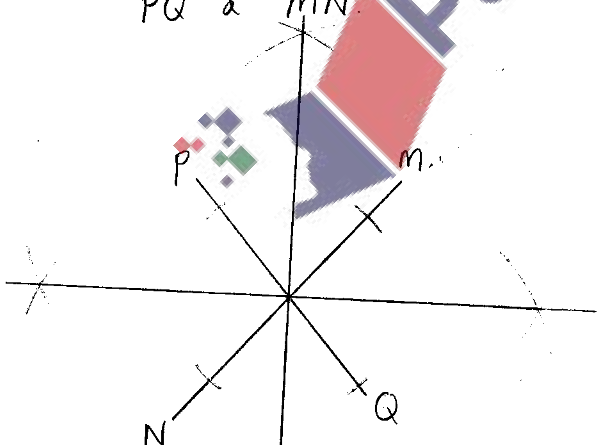
c) Equidistant from 2 given point

Example: Construct a locus that is equidistant from point P & point Q



d) Equidistant from 2 given intersecting lines

Example: Construct a locus equidistant from PQ & MN



(18) Mensuration

a) Circumference of circle = $2\pi r$

b) Area of circle = πr^3

c) Area of parallelogram = Length × ⊥ height

d) Area of trapezium = $\frac{1}{2}(l_1 + l_2) \times$ height

e) Volume of a cuboid = $l \times b \times h$

f) Volume of prism = surface area × height

g) Volume of cylinder = $\pi r^2 h$

h) Surface area of cuboid = $2(lb) + 2(bh) + 2(lh)$

i) Surface area of cylinder = $2\pi r^2 + 2\pi r h$

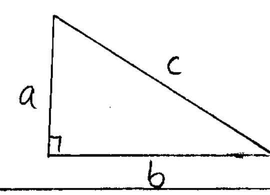
j) Arc length = $\frac{\theta}{360} \times 2\pi r$

k) Area of sector = $\frac{\theta}{360} \times \pi r^2$

(19) Trigonometry

Right-angled triangle

a) $\boxed{a^2 + b^2 = c^2}$

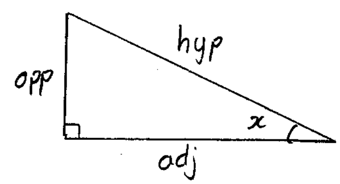


b) TOA CAH SOH

$\tan x = \frac{opp}{adj}$

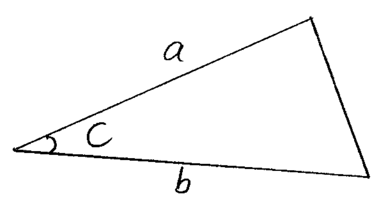
$\cos x = \frac{adj}{hyp}$

$\sin x = \frac{opp}{hyp}$



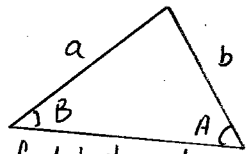
Not a right-angled triangle

c) Area of Δ = $\frac{1}{2} a b \sin C$



d) Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$



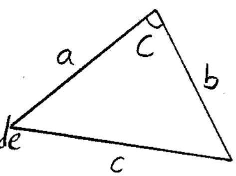
When to use?

① Given 2 side & 1 angle, find last angle

② Given 2 angle & 1 side, find last side

e Cosine Rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$



When to use?

① Given 2 sides & 1 angle, find last side

② Given 3 sides, find angle

⑳ Statistics

a) Mode, median & mean

Example: normal die, numbered 1 to 6, rolled 50 times

Score	1	2	3	4	5	6
Frequency	15	10	7	5	6	7

Mode = 15 ⇒ score with highest frequency

median ⇒ score in the middle position

$$\frac{50+1}{2} = 25.5 \Rightarrow 25^{th} \text{ \& } 26^{th} \text{ position}$$

($25^{th} \Downarrow 2$, $26^{th} \Downarrow 3$)

$$\text{median} = \frac{2+3}{2} = 2.5$$

$$\text{Mean} = \frac{\text{total score}}{\text{frequency}}$$

$$= \frac{1\times15 + 2\times10 + 3\times7 + 4\times5 + 5\times6 + 6\times7}{50}$$

$$= 2.96$$

b) For histogram,

$$\text{Frequency density} = \frac{\text{frequency}}{\text{width}}$$

㉑ Probability

If we call a particular event 'A' then the probability of 'A' happening is

$$P(A) = \frac{\text{Number of different way A can happen}}{\text{Total number of outcomes}}$$

The 'and' rule

$$P(A \text{ and } B) = P(A) \times P(B)$$

The 'or' rule

$$P(A \text{ or } B) = P(A) + P(B)$$

㉒ Matrice

For a matrix, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

determinant $A = ad - bc$

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Example: Find A^{-1} of $A = \begin{bmatrix} -6 & 7 \\ -4 & 3 \end{bmatrix}$

$$A^{-1} = \frac{1}{(-6)(3)-(7)(-4)}\begin{bmatrix} 3 & -7 \\ 4 & -6 \end{bmatrix}$$

$$= \frac{1}{-18+28}\begin{bmatrix} 3 & -7 \\ 4 & -6 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 3 & -7 \\ 4 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{10} & -\frac{7}{10} \\ \frac{2}{5} & -\frac{3}{5} \end{bmatrix}$$

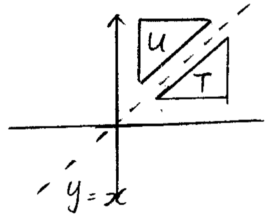
(23) Transformation

a) Reflection

Example : Describe transformation T to U

Reflection [1 mark]

$y = x$ [1 mark]



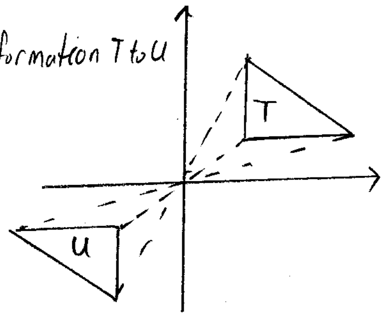
b) Rotation

Example : Describe transformation T to U

Rotation [1 mark]

Centre (0,0) [1 mark]

180° [1 mark]

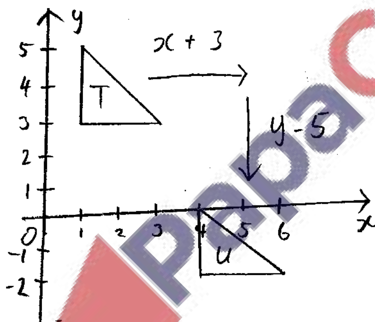


c) Translation

Example : describe the transformation T to U

Translation [1 mark]

$\begin{pmatrix} 3 \\ -5 \end{pmatrix}$ [1 mark]



d) Enlargement

Example : describe the transformation T to U

Enlargement [1 mark]

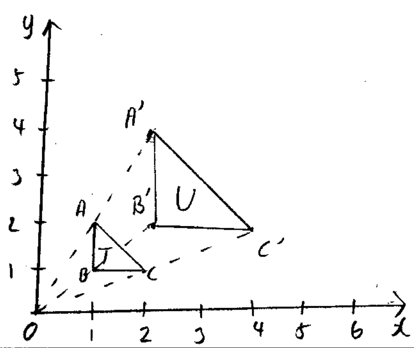
Centre (0,0) [1 mark]

⇒ Draw 2 lines AA' & BB' interception is the centre

scale factor $= \frac{OA'}{OA}$

$= \frac{4}{2}$

$= 2$



e) Shearing

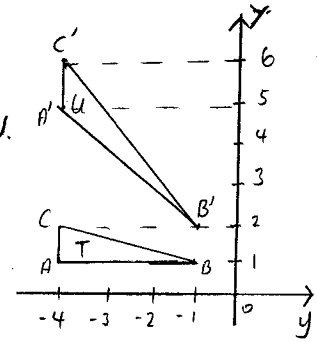
Example :

Describe the transformation T to U.

Shearing [1 mark]

Invariant line, y-axis [1 mark]

Shear factor = -1



How to find invariant line?

① Draw 2 lines AB & A'B' & find interception point 1

② Draw 2 lines CB & C'B' & find interception point 2.

③ Connect interception point 1 & 2 to get invariant line.

How to find shear factor?

shear factor $= \frac{\text{distance from invariant to old pt}}{\text{distance old point to new pt.}}$

Note : dist to the left ⇒ -ve

" " " right ⇒ +ve

dist upward ⇒ +ve

dist downward ⇒ -ve

f) Stretching

Example : Describe transformation T to U.

Stretching [1 mark]

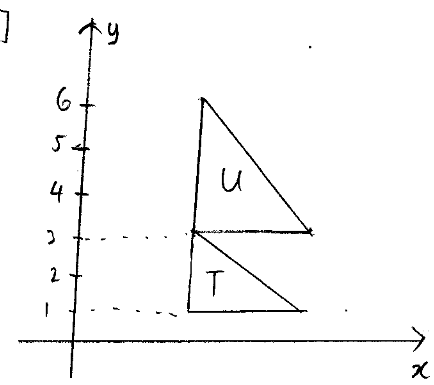
invariant [1 mark]
x-axis

stretch factor

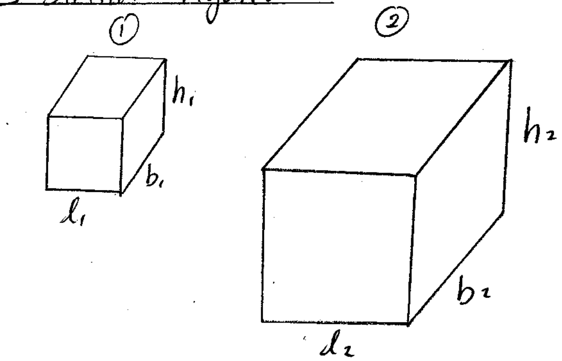
$= \frac{\text{invariant to new pt}}{\text{invariant to old pt}}$

$= \frac{6}{3}$

$= 2$



(24) Similar Figures.



a) $\left(\frac{l_1}{l_2}\right) = \left(\frac{b_1}{b_2}\right) = \left(\frac{h_1}{h_2}\right)$

b) $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$

c) $\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$